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The Electric Charge Confining in Polyakov's Compact QED in R^4

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Abstract We present a Wilson Loop evaluation of the binding energy of electric static charges in A.M. Polyakov's Compact QED in R^4 through path integrals.

Keywords Wilson loops · Compact QED

1 Introduction

It has been argued by A.M. Polyakov [1] that the presence of monopoles excitations in the vacuum of Compact Quantum Electrodynamics is the main fact responsible for the confinement or screening of the electric charge of the electron excitations.

In this short note, we intend to show such result by elementary manipulations with abelian Wilson loops path integrals in the framework of dimensional regularization and Polyakov's proposal for phenomenologically represent the dynamics of the theory's monopole condensates through an effective dynamics of a low-energy rank-two antisymmetric tensor field [2] (see Appendix for some comments).

Let us thus start our analyzes by recalling the Polyakov's propose to represent the compact QED monopole dynamics by means of the path integral for a rank-two antisymmetric tensor field [2]

$$\begin{aligned} Z &= \int D^F[B_{\mu\nu}]\exp\left\{-\frac{1}{4e^2}\int d^4x\left[B^2_{\mu\nu} + dB\ \text{arsen}\left(\frac{dB}{m^2}\right) + m^2\sqrt{1-\left(\frac{dB}{m^2}\right)^2}\right](x)\right\} \\ &\overset{\text{low-energy}}{\sim} \int D^F[B_{\mu\nu}]\exp\left\{-\frac{1}{4e^2}\int d^4x(B_{\mu\nu}[-(\partial^2)+m^2]B_{\mu\nu})(x)\right\} \end{aligned} \tag{1}$$

where m^2 is a dimensional transmutation parameter which is (phenomenologically) signaling the energy scale where theory's non-perturbative effects are expected to be relevant.

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In order to consider the presence of the electromagnetic field dynamics we consider an interaction with the electromagnetic field proposed as in [3]. Namely

$$\begin{aligned} Z = \int D^F[B_{\mu\nu}]D^F[A_\mu]\delta(\partial_\mu A_\mu)\exp\left(-\frac{1}{4e^2}\int d^4x F^2_{\mu\nu}(A)(x)\right) \\ \times \exp\left(-\frac{1}{4e^2}\int d^4x(B_{\mu\nu}[-(\partial^2)+m^2]B_{\mu\nu})(x)\right) \\ \times \exp\left(\theta\int d^4x(B^*_{\mu\nu}F^{\mu\nu}(A))(x)\right). \end{aligned} \tag{2}$$

Let us show the electric charge confinement in the effective theory as described by the path integral (2) under the constraint of the fine tunning value $\theta = |m|$.

In order to analyze such phenomenon, we consider the binding energy between two probing electric static charges with opposite charge's signal with a space-time trajectory description $C_{(R,T)}$, the loop boundary of the rectangle $S_{(R,T)} = \{-\frac{T}{2} \le x^0 \le +\frac{T}{2};\ -\frac{R}{2} \le x^1 \le +\frac{R}{2}\}$.

We have thus the standard result for this binding energy in terms of the gauge invariant Wilson loop

$$E_{\text{bin}}(R) = \lim_{T\to\infty}\left\{-\frac{1}{T}lg\overbrace{\left\langle\exp\left(i\int_{C_{(R,T)}} A_\mu(X^\alpha(s))dX^\mu(s)\right)\right\rangle}^{W[C_{(R,T)}]}\right\} \tag{3}$$

where the euclidean path integral average, normalized to unity, is defined by (2). After realizing the $B_{\mu\nu}(x)$ Gaussian path integral, we arrive at the effective $U(1)$-invariant path integral result for the Wilson Loop observable inside (3).

$$\begin{aligned} \langle W[C_{(R,T)}]\rangle = \int D^F[A_\mu]\delta^{(F)}(\partial_\mu A_\mu) \\ \times \exp\left\{-\frac{1}{2e^2}\int d^4x d^4y A_\mu(x)[(-\partial^2)_x\delta^{(4)}(x-y)\right. \\ \left. -\theta^2(-\partial^2)_x(-\partial^2_x+m^2)^{-1}(x,y)]A^\mu(y)\right\} \\ \times \exp\left(i\int_{C_{(R,T)}} A_\mu(X^\alpha(s))dX^\mu(s)\right). \end{aligned} \tag{4}$$

The searched binding energy takes thus the simple form in a dimensional regularized integral-distributional form

$$E_{\text{bin}}(R) = \lim_{T\to\infty}\left\{-\frac{e^2}{T}\int\frac{d^\nu k}{(2\pi)^\nu}f_\mu(k,C_{(R,T)})D(k^2)f^\mu(-k,C_{(R,T)})\right\} \tag{5}$$

with the rectangle form factors [4]

$$f_\mu(k,C_{(R,T)}) = \oint_{C_{(R,T)}} e^{-i(k_0x_0(s)+k_1x_1(s))}\frac{dx_\mu(s)}{ds}$$

$$= \iint_{S_{(R,T)}} \varepsilon^{\mu\nu} \left[\frac{\partial}{\partial x_\nu} (e^{-ik_\alpha x_\alpha}) \right] d^2 x_\alpha$$

$$= \begin{cases} -\frac{4}{k_0} \sin(\frac{k_0 T}{2}) \sin(\frac{k_1 R}{2}), & \text{for } \mu = 0, \\ +\frac{4}{k_1} \sin(\frac{k_0 T}{2}) \sin(\frac{k_1 R}{2}), & \text{for } \mu = 1. \end{cases} \tag{6}$$

The effective electromagnetic propagator, local in momentum space, is explicitly given by

$$D(k^2) = \left[k^2 - \frac{\theta^2 u^2}{k^2 + m^2} \right]^{-1} = \frac{k^2 + m^2}{k^4 + k^2(m^2 - \theta^2)}. \tag{7}$$

Now one can easily see that for the fine tunning choice $m = +\theta$, one obtains after straightforwardly calculations [4], the expected confining Cornell form [5] for the binding energy

$$E_{\text{bind}}(R) = \left(-\frac{e^2}{4\pi R} \right) + ((4\pi e^2 m^2) R) \tag{8}$$

it is worth that for $m \neq \theta$, or $\theta = i\overline{\theta}$ ($\overline{\theta} \in R$), to call the reader attention that the binding energy is of the Yukawa form and thus leading to a screening picture for the inter-electronic potential for compact *QED* in R^4. All this mean that some sort of "lethal" instability with the usual wisdom of confining in compact *QED* may be hidding in the previous studies in the subject [1, 3].

At this point, let us comment and compare ours results with the attempts done in [3] through Hamiltonian methods.

We feel that these works may be not well-defined by the somewhat cumbersome use of cut-offs (otherwise infinite) ordinary integrals. For instance, in [5], it has been obtained the following expression for the binding energy (see (17)–(29) of [3] where $R = (\vec{y} - \vec{y}')$, $M^2 = m^2 + e^2$ in the refs author's notation):

$$E_{\text{bind}}(R) = V^{(1)}_{(R)} + V^{(2)}_{(R)} \tag{9}$$

with

$$V^{(1)}_{(R)} = -\frac{e^2}{2} \int d^3x \int_{\vec{y}}^{\vec{y}'} dz'_i \delta^{(3)}(x - z') \left(\frac{1}{\nabla_x^2 - M^2} \nabla_x^2 \right) \left(\int_{\vec{y}}^{\vec{y}'} dz^i \delta^{(3)}(x - z) \right)$$
$$= (e^2 e^{-MR})/4\pi R \tag{10}$$

and

$$V^{(2)}(R) = +\frac{e^2 m^2}{2} \int d^3x \int_{\vec{y}}^{\vec{y}'} dz'_i \delta^{(3)}(x - z') \left(\frac{1}{\nabla_x^2 - M^2} \right) \int_{\vec{y}}^{\vec{y}'} dz^i \delta^{(3)}(x - z)$$
$$= -\frac{e^2}{4\pi} \left(e^{-MR}/R \right) + \frac{e^2 M}{8\pi} R \ell n \left(1 - \frac{\varepsilon^3}{M^2} \right) \tag{11}$$

where ε is a cutt-off to be imposed in order to make sense for their divergent ordinary integrals. However, it appears that this process of handling distributions has drawbacks, since one can easily use the screening result (10) to evaluate (11) through the use of the simple operatorial decomposition

$$
\begin{aligned}
V^{(2)}(R) &= \frac{e^2m^2}{2}\int d^3x \int_{\vec{y}}^{\vec{y}'} dz_i' \delta^{(3)}(x-z')\left(\frac{(\nabla_x)^2}{(\nabla_x)^2(\nabla_x^2-M^2)}\right) \\
&\quad \times \left(\int_{\vec{y}}^{\vec{y}'} dz^i \delta^{(3)}(x-z)\right) \\
&= \frac{e^2m^2}{2}\left\{\int d^3x \int_{\vec{y}}^{\vec{y}'} dz_1^i \delta^{(3)}(x-z')\left[-\frac{1}{M^2}\frac{1}{(\nabla_x)^2}+\frac{1}{M^2}\frac{1}{(\nabla_x^2-M^2)}\right]\right. \\
&\quad \left.\times \left(\int_{\vec{y}}^{\vec{y}'} dz^i \delta^{(3)}(x-z)\right)\right\} \\
&= -\frac{e^2M^2}{2M^2}\left(\frac{1}{R}\right)+\frac{e^2m^2}{2M^2}\left(\frac{e^{-MR}}{4\pi R}\right)
\end{aligned} \tag{12}
$$

which clearly is finite and differs from the their claimed cut-off implicit confining result. At this point let us suggest that the use of Hamiltonian formalism to handle mathematically gauge theories, specially Yang-Mills theory and its variants is somewhat difficulted by the fact that one must fix the gauge and this makes recourse for formal BRST realizations of Gauge Invariance in the observables evaluation in order to make the results physically acceptable. Certainly such somewhat formal technique making use of "infinitesimal" grassmanian parameters in a theory with solely physical bosoms fields as imputs must be applied very carefully outside the well-founded evaluations of matrix (on-sheel) scattering amplitudes which does not exist in pure Yang-Mills theory due to severe infrared divergencies. That was the main reason of reformulating all the gauge theories by means of gauge invariant path integrals and mainly with the objective that in the confining phase all the evaluations should be done non-perturbativelly (see appendix for supplementary comments).

Appendix: The Dynamics of the $QCD(SU(\infty))$ Tensor Fields From Strings

In this appendix we intend to high-light on some ideas and loop space formulae about non-space time supersymmetric random surfaces representations for bosonic $QCD(SU(N_c))$ (including the t'Hooft plannar diagrams limit of $N_c \to \infty$).

Alexandre M. Polyakov in the article [6], has argued that one possible random surface path integral representation for Wilson Loops in bosonic $QCD(SU(N_c))$ shall be given by our discovery that mathematical bosonic strings interacting with A Migdal intrinsic fermionic Strings degree of freedom in the $SU(3)\times SU(2)\times U(3)$ fundamental representation (in an non-abelian bosonized form) and interacting with a rank-two antisymmetric tensor field through the closed string world-sheet orientation tangent tensor namely [6–12].

$$
\begin{aligned}
W[C] = \Bigg\langle \sum_{S_i\,\partial s=\ell}\sum_{\{g\}}\Bigg\{ &\exp\left[-\int d^2\xi\, \zeta^{\mu\nu}(X^\beta(\xi)).\,\mathrm{Tr}(g^{-1}B_{\mu\nu}(X(\xi)g)\right] \\
&\times \exp[-(\sigma\text{—model action for } g \text{ with} \\
&\qquad \text{Wess-Zomino-Novikov Topological Term})]\Bigg\}\Bigg\rangle_B .
\end{aligned} \tag{A.1}
$$

The author of the second set of [7–12] present arguments that the average over the $B_{\mu\nu}(x)$, probably representing the non-trivial random flux structure of the QCD-vaccum should be defined by a pure white-noise set of random fluxes. In the path integral language [6, 13], the B-average should defined by the functional measure:

$$\langle\rangle_B = \int D^F[B_{\mu\nu}(x)] \exp\left\{-\frac{1}{4(g_\infty)^2}\int d^4x B^2_{\mu\nu}(x)\right\} \tag{A.2}$$

where $g^2_\infty = \lim_{N_i\to\infty}(g^2N_c) < \infty$ denotes the $N_c = +\infty$ t'Hooft QCD coupling constant.

It is very important to remark now that (A.1) with the White-Noise random flux average (A.2), satisfies the QCD loop wave equation exactly under the geometrical hypothesis that the bosonic surfaces (string would-sheets) defining the QCD string should satisfies the self-intersecting orthogonality tangent plane constraints ($\xi = (s,\sigma)$), $X^\mu(s,0^+) = \ell^u(s)$)

$$\begin{aligned} &\zeta_{\mu\nu}(X^\beta(s,\sigma))\zeta^{\mu\nu}(X^\beta(s',\sigma')) = 0 \quad \text{if } s \neq s', \\ &\zeta^{\mu\nu}(X^\beta(s,\sigma))\zeta_{\mu\nu}(X^\beta(s',\sigma')) \neq 0 \quad \text{if } s = s'. \end{aligned} \tag{A.3}$$

Note that the second constraint in (A.3) means that one allows non-trivial topology in the intrinsic (mathematical) string time-direction evolution $0 \le \sigma < \infty$. However, the first geometrical constraint may be connected (not proved yet) to the fact that the bosonic loops $X^\mu(s,\bar\sigma)$ (with $\bar\sigma$ fixed) should physically corresponding to R^4 space-time euclidean quark-antiquark trajectories on the fermionic quark functional determinant in the presence of an abelian color singlet vectorial source, for instance:

$$\begin{aligned} \ell g \det(\not\partial(A_\mu) + J_\mu) = {}& -\frac{1}{2}\int_0^\infty \frac{d\Gamma}{T}\left\{\int d^4z^\beta \int_{X^\beta(0)=X^\beta(T)=z^\beta} D^F[X(s)]\right. \\ & \times \exp\left(-\frac{1}{2}\int_0^T ds\cdot X^2(s)\right) \times \exp\left(ie\int_0^T ds\, J_\mu(X(s))\right) \\ & \times \mathrm{Tr}\left[\mathbb{P}\mathrm{Spin}\left(\exp\frac{g^2}{4i}\int_0^T ds[\gamma^\mu,\gamma^\nu]\left[\frac{\delta}{\delta\sigma_{\mu\nu}(X(s))}\right]\right)\right] \\ & \left.\times \frac{1}{N_c}\mathrm{Tr}_{\mathrm{color}}\,\mathbb{P}\left(\left[\exp ig\int_0^T A_\mu(X(s))dX^\mu(s)\right)\right]\right\} \end{aligned} \tag{A.4}$$

with $\frac{\delta}{\delta\sigma_{\mu\nu}(x^\beta)}$ denoting the formal Mandelstam-Migdal area functional derivative $\frac{\delta}{\delta\sigma_{\mu\nu}(X^\beta(s))} = \frac{\delta}{\delta[(\dot X_\mu X_\nu - \dot X_\nu X_\mu)(s)]}$ and satisfying thus the Pauli Exclusion principle which by its turn translates geometrically by the condition that at non-trivial proper-time trajectories self-intersections $X^\mu(s) = X^\mu(s')$ $(0 < s, s' < T)$, we have always that $\frac{dX^\mu(s)}{ds}\cdot\frac{dX_\mu(s')}{ds'} \equiv 0$. This important Pauli Exclusion random surfaces probably signals that strings representations for scalar quark QCD is ill-defined in relation to strings representations for fermionic quark QCD as proposed in [7–16].

At this point one may expect that in the very low energy scale of QCD, one can integrate out all the strings degrees of freedom and arrives at the somewhat crude however local effective rank-two antisymmetric tensor field simplest action (1), but added with Abelian Wilson Loop as a sort of remnants of the full non-Abelian colour Wilson Loop phase factor. Note that the closed loops are already under a fixed proper-time parametrization, as one can see from the loop space expression for (A.4).

As an important point to be singlet out is that the Loop Space/String program for *QCD* seems much better defined if one introduces supersymetry on the quark-antiquark world lines and string (spinning) world sheets to handle directly the Lorentz spin content of the quarks, instead of the obligatory operational loop Feynman Lorentz spin factor inside (A.4) with the operational expression:

$$\Phi^{\mathrm{Spin}}[\ell] = \mathbb{P}_{\mathrm{Dirac}}\left\{\exp\left(-\frac{g_\infty^2}{2}\oint_C ds[\gamma^\mu,\gamma^\nu]\times\left[\frac{\delta}{\delta\zeta_{\mu\nu}(X^\beta(s,\sigma))}\right]_{\sigma\to 0^+}\right)\right\} \tag{A.5}$$

equivalently written by the introduction of a new set of intrinsic neutral Fermion fields in the string world sheet: $\{\psi^\mu(\xi)\}_{\mu=0,1,2,3}$ and representing the "String Lorentz Spin". Namely:

$$\begin{aligned}\Phi^{\mathrm{Spin}}_{\alpha\beta}[\ell] &= \int [D^F\psi^\mu_{(\xi)}]\times\exp\left[-\frac{1}{2}\int d^2\xi(\psi^\mu(i\gamma)\psi_\mu)(\xi)\right](\psi_\alpha(0,0)\psi_\beta(T,0)\\ &\quad\times\exp\left(-\frac{g_\infty^2}{2}\int d^2\xi(\psi_\alpha[\gamma^\mu,\gamma^\nu]_{\alpha\beta}\psi_\beta)(\xi)\right.\\ &\quad\left.\times\,\mathrm{Tr}\left[g^{-1}(\xi)\frac{\delta}{\delta\zeta_{\mu\nu}(X^\beta(\xi))}g(\xi)\right]\right).\end{aligned} \tag{A.6}$$

It is hoped that some sort of solitons/Lorentz Spin Vertexs in the *QCD* Spinning, flavor charged string theory (A.1)–(A.5) should be candidates to describe the Baryons Physics, basic issue not yet handled in the loop space framework and others holographic proposals for strings representations in supersymmetric Yang-Mills theory.

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